## Product-to-Sum

 Formulas$$
\begin{aligned}
\cos (A+B)=\cos A \cos B- & \sin A \sin B \\
\cos (A-B)=\cos A \cos B+ & \sin A \sin B \\
\cos (A+B)+\cos (A-B)= & 2 \cos A \cos B \\
& \cos A \cos B=\frac{1}{2}[\cos (A-B)+\cos (A+B)] \\
\cos (A-B)-\cos (A+B)= & 2 \sin A \sin B \\
& \sin A \sin B=\frac{1}{2}[\cos (A-B)-\cos (A+B)]
\end{aligned}
$$

$$
\begin{aligned}
& \sin (A+B)=\sin A \cos B+\cos A \sin B \\
& \sin (A-B)=\sin A \cos B-\cos A \sin B \\
& \sin (A+B)+\sin (A-B)=2 \sin A \cos B \\
& \sin A \cos B=\frac{1}{2}[\sin (A+B)+\sin (A-B)] \\
& \sin A \sin B=\frac{1}{2}[\cos (A-B)-\cos (A+B)] \\
& \cos A \cos B=\frac{1}{2}[\cos (A-B)+\cos (A+B)] \\
& \sin A \cos B=\frac{1}{2}[\sin (A+B)+\sin (A-B)] \\
& \text { Product - to - Sum formulas }
\end{aligned}
$$

find exact value for $\cos 45^{\circ} \sin 15^{\circ}$

$$
\begin{aligned}
\cos 45^{\circ} \sin 15^{\circ} & =\frac{1}{2}\left[\sin \left(45^{\circ}+15^{\circ}\right)+\sin \left(45^{\circ}-15^{\circ}\right)\right] \\
& =\frac{1}{2}\left[\sin 60^{\circ}+\sin 30^{\circ}\right] \\
& =\frac{1}{2}\left[\frac{\sqrt{3}}{2}+\frac{1}{2}\right]=\frac{1}{2} \cdot \frac{1}{2}[\sqrt{3}+1] \\
& =\frac{1}{4}(\sqrt{3}+1)
\end{aligned}
$$

find exact value for $4 \sin \left(x+\frac{\pi}{4}\right) \sin \left(x-\frac{\pi}{4}\right)$

$$
\begin{aligned}
4 \sin \left(x+\frac{\pi}{4}\right) \sin \left(x-\frac{\pi}{4}\right) & =4 \cdot \frac{1}{2}[\cos (\underbrace{-2}_{\frac{\pi}{2}})-\cos (\underbrace{+}_{2 x})] \\
x+\frac{\pi}{4}-\left(x-\frac{\pi}{4}\right)=\frac{\pi}{2} & =2\left[\cos \frac{\pi}{2}-\cos 2 x\right] \\
x+\frac{\pi}{4}+x-\frac{\pi}{4}=2 x & =2[0-\cos 2 x] \\
\cos \frac{\pi}{2}=\cos 90^{\circ}=0 & =-2 \cos 2 x
\end{aligned}
$$

Evaluate $\operatorname{Cos} \frac{5 \pi}{12} \operatorname{Cos} \frac{\pi}{12}$

$$
\begin{aligned}
& \cos \frac{5 \pi}{12} \cos \frac{\pi}{12}=\frac{1}{2}[\cos (\underbrace{-}_{\pi / 3})+\cos (\underbrace{+}_{\frac{\pi}{2}})] \\
& \begin{aligned}
\frac{5 \pi}{12}-\frac{\pi}{12}=\frac{4 \pi}{12}=\frac{\pi}{3} & =\frac{1}{2}\left[\cos \frac{\pi}{3}+\cos \frac{\pi}{2}\right] \\
\frac{5 \pi}{12}+\frac{\pi}{12}=\frac{6 \pi}{12}=\frac{\pi}{2} & =\frac{1}{2}\left[\frac{1}{2}+0\right] \\
& =\frac{1}{4}
\end{aligned}
\end{aligned}
$$

